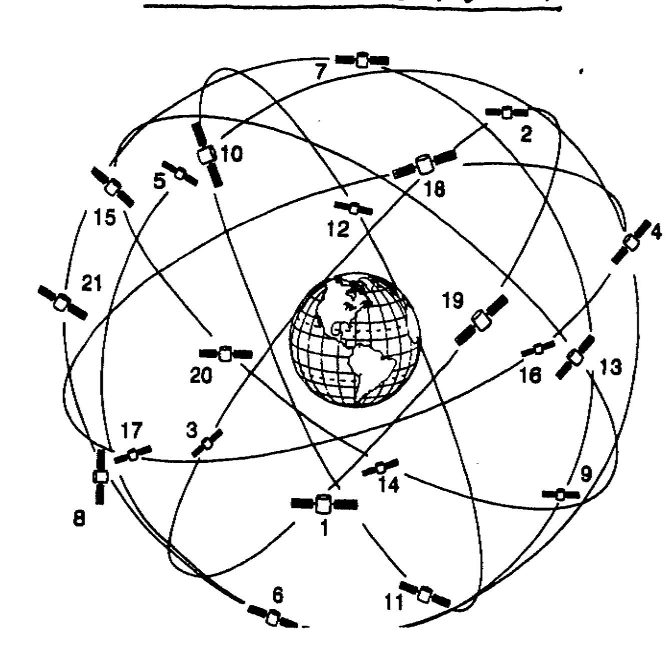
Global Positioning System

The GPS constallation consists of

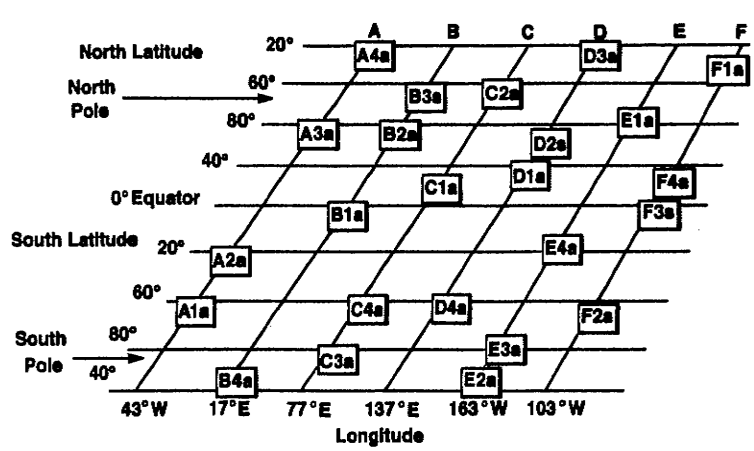
- 24 satellites (MEO)
- in six near-circular orbits
- at an altitude of approximately 20,000 km (11,400 miles)
- ascending nodes of the orbits separated by 60°
- The inclination of each orbit is 55°
- Therefore their are 6 circular orbits with 4 satellites per orbit
- Separation, 0°, 60°, 120°, 180°, 240°, 300° corresponds with, 43°W, 17°E, 77°E, 137°E, 163°W, 103°W (see below)

Count the orbits (6)
Count the # of satellites per orbit (4)

GPS Constellation



Satellite relative positions



How the GPS Works

- The GPS system works by determining how long it takes a radio signal transmitted from a satellite to reach a land-based receiver and, then, using that time to calculate the distance between the satellite and the earth station receiver

- Each of the 24 satellites must update its orbital elements (coordinates) continually because they vary as the satellite orbits earth
- Each satellite broadcasts its orbital elements to the receiver (user) on earth from which the receiver ~~can~~ calculates its position (longitude and latitude). [These orbital element parameters are continually updated by a master control station which transmits them up to the satellites]
- The receiver must receive orbital elements signals from Three satellites to pinpoint its longitude and latitude
- Therefore Three equations from three satellites are required to solve for the three unknown coordinates
- To eliminate the clock bias error, a fourth ~~equation~~ satellite is needed to produce the fourth equation necessary to solve for four unknowns using simultaneous equations. [See the next slide (pu90)]
- Each satellite carries its own atomic clock

Coordinate Systems

Several coordinate systems are used by Astrodynamics, but only some are useful for our requirements

- Heliocentric - ecliptic coordinate system
- Geocentric - equatorial coordinate system
- Right ascension - declination coordinate system
- Perifocal coordinate system
- Celestial horizon coordinate system

used with the GPS system

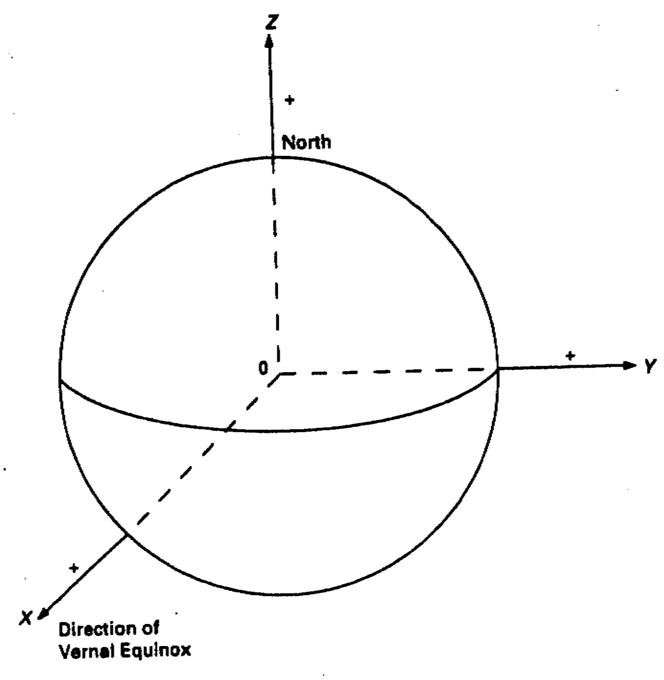


Figure 2.2 Geocentric–equatorial coordinate system.

Geocentric–equatorial coordinate system

The geocentric–equatorial coordinate system is shown in figure 2.2. Here the equator is the fundamental plane; the geocentre constitutes the origin; the positive *X*-axis is in the direction of the Vernal Equinox, the positive *Y*-axis to the east of the Vernal Equinox, and the *Z*-axis points in the direction of the North Pole.

In order to completely describe the position of a satellite in a three-dimensional space at any given time, a classical set of six parameters, called the orbital parameters, is used. These parameters, shown in figures 2.8 and 2.9, are briefly described below. Sometimes these parameters may be substituted by others but it should always be possible to convert them into one of the following sets:

Orbital Parameters
≡ Orbital Elements

Note that there are six orbital elements (sometimes referred to as the Keplerian element set)

1. The *semi-major* axis, a, describes the size of a conic orbit (ellipse or circle) for satellite communications (figure 2.8).
2. The *eccentricity*, e, shows the ellipticity of the orbit (figure 2.8).
3. The *inclination*, i, is the angle between the plane of the orbit and the equatorial plane measured at the ascending node in a northward direction. An ascending node is the point at which the satellite crosses the equatorial plane moving from south to north. Similarly a descending node is the point where the satellite crosses the equatorial plane moving in the direction from north to south. The line joining these two nodes is called the line of nodes (figure 2.9).
4. The *right ascension* of an ascending node, Ω, is the angle between the X-axis (the direction of Vernal Equinox) and the ascending node (figure 2.9).
5. The *argument of perigee*, ω, is the angle in the orbital plane between the line of nodes and the perigee of the orbit (figure 2.9).
6. Time, t_p, is the time elapsed since the satellite passed the perigee (figure 2.8). Sometimes this parameter is given as *mean anomaly* M. The parameter t_p can be readily calculated from M (see next section).

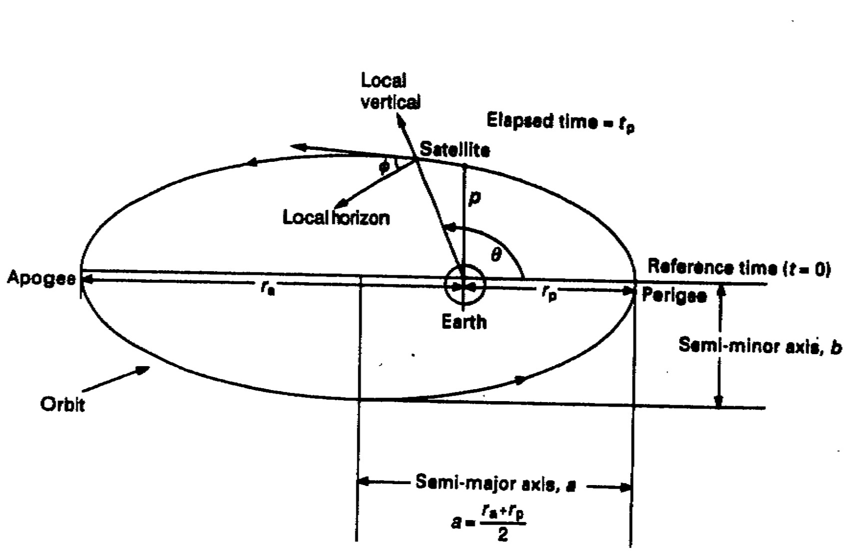
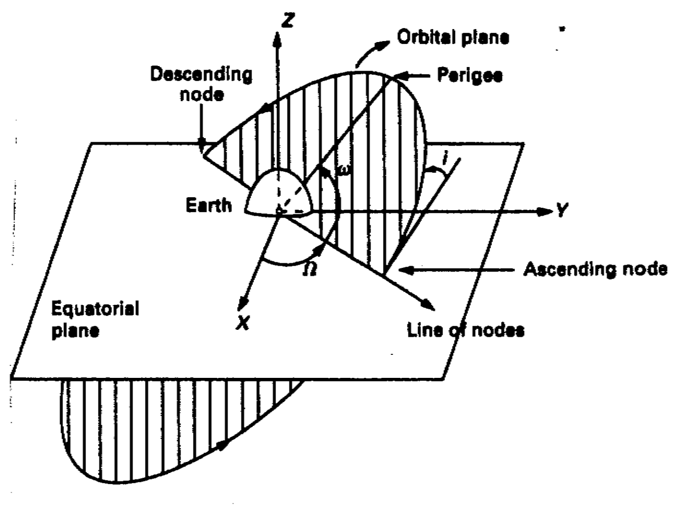
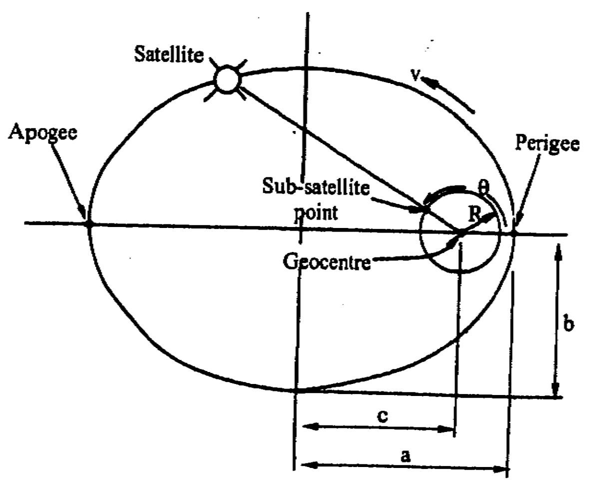


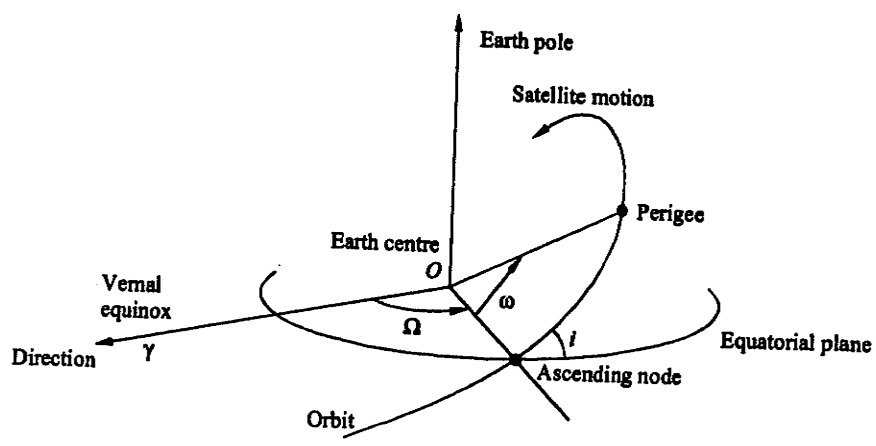
Figure 2.8 Major parameters of an elliptical orbit.



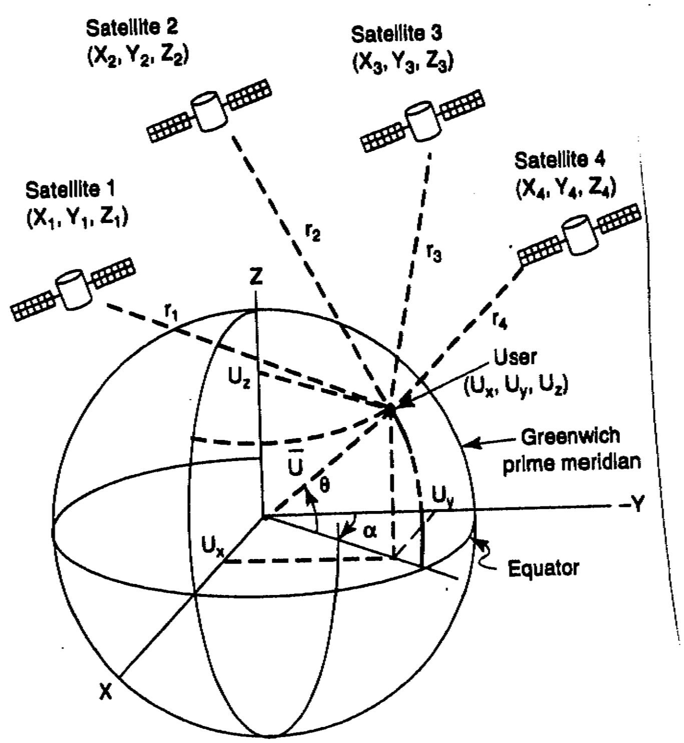
Orbital parameters in a geocentric–equational coordinate system



a: semi-major axis
b: semi-minor axis
e: eccentricity = $(1 - (b/a)^2)^{0.5}$; $0 \le e < 1$; (circle: $e = 0$)
c: distance between centre of ellipse and focal point = ae
R: mean radius of Earth
r, θ polar coordinates of satellite; θ (the true anomaly) is measured from perigee



Solutions to the simultaneous equations for determining latitude and longitude



$$(X_1 - U_x)^2 + (Y_1 - U_y)^2 + (Z_1 - U_z)^2 = (r_1 - C_b)^2$$

$$(X_2 - U_x)^2 + (Y_2 - U_y)^2 + (Z_2 - U_z)^2 = (r_2 - C_b)^2$$

$$(X_3 - U_x)^2 + (Y_3 - U_y)^2 + (Z_3 - U_z)^2 = (r_3 - C_b)^2$$

$$(X_4 - U_x)^2 + (Y_4 - U_y)^2 + (Z_4 - U_z)^2 = (r_4 - C_b)^2$$

$$\text{User's latitude} = \theta = \mathrm{COS}^{-1} \frac{\sqrt{U_x^2 + U_y^2}}{|U|}$$

$$\text{User's longitude} = \alpha = \mathrm{TAN}^{-1} \frac{U_x}{U_y}$$

GPS satellite position calculations

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#3

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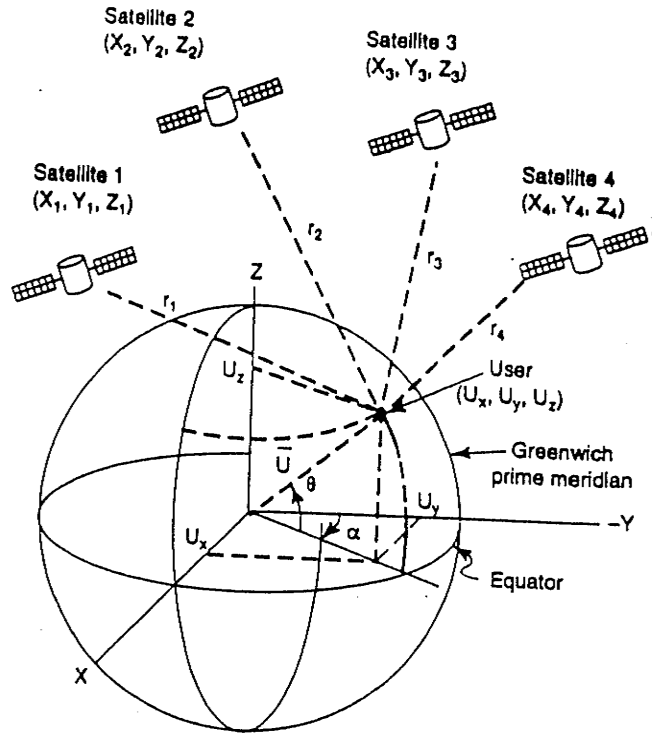

- receiver position is, (U_x, U_y, U_z)
- The four satellites have positions, (X_i, Y_i, Z_i)
- The earth station Location is at $(\emptyset, \emptyset, 6378\,km)$ (U_x, U_y, U_z)

$$(X_1 - U_x)^2 + (Y_1 - U_y)^2 + (Z_1 - U_z)^2 = (r_1 - C_b)^2$$
$$(X_2 - U_x)^2 + (Y_2 - U_y)^2 + (Z_2 - U_z)^2 = (r_2 - C_b)^2$$
$$(X_3 - U_x)^2 + (Y_3 - U_y)^2 + (Z_3 - U_z)^2 = (r_3 - C_b)^2$$
$$(X_4 - U_x)^2 + (Y_4 - U_y)^2 + (Z_4 - U_z)^2 = (r_4 - C_b)^2$$

User's latitude $= \theta = \text{COS}^{-1} \dfrac{\sqrt{U_x^2 + U_y^2}}{|U|}$

User's longitude $= \alpha = \text{TAN}^{-1} \dfrac{U_x}{U_y}$

From equation 12.1, page 465

The measured distance to satellite i is called a psuedorange, $PR_i = r_i$

Psuedorange = (propagation time delay T_i)(velocity of light)

$$PR_i = r_i = (T_i)(c)$$

but, $r_1 = r_2 = r_3 = r_4 = r$ and, $T_1 = T_2 = T_3 = T_4 = T$

$$r = (T)(c)$$
$$= (0.17097528\ \text{seconds})(2.99792458\ \text{meter/seconds})$$
$$= 51{,}257.099\ km$$

From equation 12.3, page 465
(Note all distances are in KM)

$$(0-0)^2+(13280.5-0)^2+(23002.5-6378.0)^2=(51{,}257.099-\tau c)^2$$

$$(0-0)^2+(-13280.5-0)^2+(23002.5-6378.0)^2=(51{,}257.099-\tau c)^2$$

$$(13280.5-0)^2+(0-0)^2+(23002.5-6378.0)^2=(51{,}257.099-\tau c)^2$$

$$(-13280.5-0)^2+(0-0)^2+(23002.5-6378.0)^2=(51{,}257.099-\tau c)^2$$

Each of these four equations gives the same result

$$(13280.5)^2+(23002.5-6378.0)^2=(51{,}257.099-\tau c)^2$$

$$21{,}277.821=(51{,}257.099-\tau c)$$

Therefore, $\tau c = 29{,}979.278\ KM$

<u>Note:</u> In our drawing $C_b = \tau c$ = clock bias error

$$\tau = \frac{29{,}979.278\,KM}{c} = \frac{29{,}979.278\,KM}{2.99792458(10^8)\ \text{Meters/second}}$$

$$\boxed{\tau = 100.000010\ \text{mseconds}}$$

where τ = Clock offset error = Clock bias

Now, what does this tell us? How do we interpret these numbers?

- This Clock error is the clock error of the receiver clock relative to the GPS Satellite clock
- This clock error (100.000010 mseconds) caused an error of our distance measurement of 29,979.278 KM. What this means is that the position measurement received by the earth station will be off by this amount

- How do we remove this time error? With a fourth satellite. This is the reason why 4 satellites are needed. Accurate position measurements are made by using this fourth satellite.

- With this 4th satellite C/A code receivers can synchronize their internal clocks to GPS time within $170(10^{-9})$ seconds, corresponding to a distance measurement uncertainty of 50 meters.

Compare this 50 meters with our error of 29,979.278 km !!!